There are 5 reactions and 4 species in the OMM model.

The stoichiometric matrix is S =

```
  -1   1   1  0   0
  -1   1   0  1   0
   1  -1  -1  0   0
   0   0   1  0  -1
```

The vector of reaction velocities is v, where

```
  v(1) = k1*x1*x2
  v(2) = k2*x3
  v(3) = k3*x3
  v(4) = k4
  v(5) = k5*x4
```

The vector of mass balance equations is xdot = S*v, where

```
  xdot(1) = k2*x3 + k3*x3 - k1*x1*x2
  xdot(2) = k4 + k2*x3 - k1*x1*x2
  xdot(3) = k1*x1*x2 - k3*x3 - k2*x3
  xdot(4) = k3*x3 - k5*x4
```

We would like to define a map psi_p such that psi_p(k4) is in P. To do so we introduce a pseudospecies x5_hat=1 and let v(4) = k4*x5_hat. This gives

```
  v(1) = k1*x1*x2
  v(2) = k2*x3
  v(3) = k3*x3
  v(4) = k4*x5_hat
  v(5) = k5*x4
```

Let the map psi_p be given by

```
  k1      |-->  y1
  k2      |-->  y2
  k3      |-->  y3
  k4      |-->  p5
  k5      |-->  y4
  x1      |-->  p1
  x2      |-->  p2
  x3      |-->  p3
  x4      |-->  p4
  x5_hat  |-->  y5
```

This results in a linear velocity vector psi_p(v), where

```
  psi_p(v(1)) = p1*p2*y1
  psi_p(v(2)) = p3*y2
  psi_p(v(3)) = p3*y3
  psi_p(v(4)) = p5*y5
  psi_p(v(5)) = p4*y4
```

We can express psi_p(v) as the product P*y, where y is the vector [y1,...,y5]^T and P =

```
  p1*p2   0   0   0   0
      0  p3   0   0   0
      0   0  p3   0   0
      0   0   0   0  p5
      0   0   0  p4   0
```

From this we calculate the coefficient matrix, C = S*P =

```
  -p1*p2   p3   p3    0   0
  -p1*p2   p3    0    0  p5
```

```
   p1*p2  -p3  -p3    0   0
       0    0   p3  -p4   0
```

C is row equivalent to the reduced matrix Crref =

```
  1  -p3/(p1*p2)  0  0  -p5/(p1*p2)
  0            0  1  0       -p5/p3
  0            0  0  1       -p5/p4
  0            0  0  0            0
```

The null space of C is spanned by the columns of N =

```
  p3/(p1*p2)  p5/(p1*p2)
           1           0
           0       p5/p3
           0       p5/p4
           0           1
```

Let ybar = N*q, where q is given by

```
  q(1) = q1
  q(2) = q2
```

This gives

```
  ybar(1) = (p3*q1 + p5*q2)/(p1*p2)
  ybar(2) = q1
  ybar(3) = (p5*q2)/p3
  ybar(4) = (p5*q2)/p4
  ybar(5) = q2
```

From ybar we construct the composite forward map psi_py :

```
  k1      |-->  (p3*q1 + p5*q2)/(p1*p2)
  k2      |-->  q1
  k3      |-->  (p5*q2)/p3
  k4      |-->  p5
  k5      |-->  (p5*q2)/p4
  x1      |-->  p1
  x2      |-->  p2
  x3      |-->  p3
  x4      |-->  p4
  x5_hat  |-->  q2
```

The steady state reaction velocity vector vbar is given by psi_py(v), where

```
  vbar(1) = p3*q1 + p5*q2
  vbar(2) = p3*q1
  vbar(3) = p5*q2
  vbar(4) = p5*q2
  vbar(5) = p5*q2
```

To resolve the pseudospecies we require that psi_py(x5_hat)=1. In other words, q2=1. This gives

```
  k1      |-->  (p5 + p3*q1)/(p1*p2)
  k2      |-->  q1
  k3      |-->  p5/p3
  k4      |-->  p5
  k5      |-->  p5/p4
  x1      |-->  p1
  x2      |-->  p2
  x3      |-->  p3
  x4      |-->  p4
  x5_hat  |-->  1
```

and

```
  vbar(1) = p5 + p3*q1
  vbar(2) = p3*q1
  vbar(3) = p5
  vbar(4) = p5
  vbar(5) = p5
```

We may now proceed with the inverse substitution.

The mapping function psi_q^-1 is given by

```
  q1  |-->  y2
```

The composite inverse map psi_qp^-1:

```
  p1  |-->  x1
  p2  |-->  x2
  p3  |-->  x3
  p4  |-->  x4
  p5  |-->  k4
  q1  |-->  k2
```

The complete steady state map psi_ss is therefore

```
  k1      |-->  (k4 + k2*x3)/(x1*x2)
  k2      |-->  k2
  k3      |-->  k4/x3
  k4      |-->  k4
  k5      |-->  k4/x4
  x1      |-->  x1
  x2      |-->  x2
  x3      |-->  x3
  x4      |-->  x4
  x5_hat  |-->  1
```

The unsubstituted steady state reaction velocity vector vbar = psi_ss(v) is given by

```
  vbar(1) = k4 + k2*x3
  vbar(2) = k2*x3
  vbar(3) = k4
  vbar(4) = k4
  vbar(5) = k4
```

Verify steady state. xdot = S * vbar, where

```
  xdot(1) = 0
  xdot(2) = 0
  xdot(3) = 0
  xdot(4) = 0
```