There are 5 reactions and 4 species in the OMM model.

The stoichiometric matrix is S =

```
  -1   1   1  0   0
  -1   1   0  1   0
   1  -1  -1  0   0
   0   0   1  0  -1
```

In this variant of the OMM model, we assume the rate of product formation is proportional to the square of the enzyme-substrate complex:

```
  v(3) = k3*x3^2
```

The resulting vector of reaction velocities is v, where

```
  v(1) = k1*x1*x2
  v(2) = k2*x3
  v(3) = k3*x3^2
  v(4) = k4
  v(5) = k5*x4
```

The vector of mass balance equations is xdot = S*v, giving

```
  xdot(1) = k2*x3 + k3*x3^2 - k1*x1*x2
  xdot(2) = k4 + k2*x3 - k1*x1*x2
  xdot(3) = k1*x1*x2 - k3*x3^2 - k2*x3
  xdot(4) = k3*x3^2 - k5*x4
```

We would like to define a map psi_p such that psi_p(x3) is in set Y. To do so we introduce a pseudospecies x5_hat = x3^2 and let v(3) = k3*x5_hat. Now,

```
  v(1) = k1*x1*x2
  v(2) = k2*x3
  v(3) = k3*x5_hat
  v(4) = k4
  v(5) = k5*x4
```

Let the map psi_p be given by

```
  k1      |-->  p2
  k2      |-->  p3
  k3      |-->  p4
  k4      |-->  y5
  k5      |-->  p5
  x1      |-->  y4
  x2      |-->  p1
  x3      |-->  y1
  x4      |-->  y3
  x5_hat  |-->  y2
```

This results in a linear velocity vector psi_p(v), where

```
  psi_p(v(1)) = p1*p2*y4
  psi_p(v(2)) = p3*y1
  psi_p(v(3)) = p4*y2
  psi_p(v(4)) = y5
  psi_p(v(5)) = p5*y3
```

We can express psi_p(v) as the product P*y, where y is the vector [y1,...,y5]^T and P =

```
   0   0   0  p1*p2  0
  p3   0   0      0  0
   0  p4   0      0  0
   0   0   0      0  1
   0   0  p5      0  0
```

From this we calculate the coefficient matrix, C = S*P =

```
   p3   p4    0  -p1*p2  0
   p3    0    0  -p1*p2  1
  -p3  -p4    0   p1*p2  0
    0   p4  -p5       0  0
```

C is row equivalent to the reduced matrix Crref =

```
  1  0  0  -(p1*p2)/p3   1/p3
  0  1  0            0  -1/p4
  0  0  1            0  -1/p5
  0  0  0            0      0
```

The null space of C is spanned by the columns of N =

```
  (p1*p2)/p3  -1/p3
           0   1/p4
           0   1/p5
           1      0
           0      1
```

Let ybar = N*q, where q is given by

```
  q(1) = q1
  q(2) = q2
```

This gives

```
  ybar(1) = -(q2 - p1*p2*q1)/p3
  ybar(2) = q2/p4
  ybar(3) = q2/p5
  ybar(4) = q1
  ybar(5) = q2
```

From ybar we construct the composite forward map psi_py :

```
  k1      |-->  p2
  k2      |-->  p3
  k3      |-->  p4
  k4      |-->  q2
  k5      |-->  p5
  x1      |-->  q1
  x2      |-->  p1
  x3      |-->  -(q2 - p1*p2*q1)/p3
  x4      |-->  q2/p5
  x5_hat  |-->  q2/p4
```

The steady state reaction velocity vector vbar is given by psi_py(v), where

```
  vbar(1) = p1*p2*q1
  vbar(2) = p1*p2*q1 - q2
  vbar(3) = q2
  vbar(4) = q2
  vbar(5) = q2
```

To resolve the pseudospecies we require that psi_py(x5_hat) = psi_py(x3^2).
In other words, (-(q2 - p1*p2*q1)/p3)^2 =q2/p4. This gives

```
  k1      |-->  p2
  k2      |-->  p3
  k3      |-->  p4
  k4      |-->  q2
  k5      |-->  p5
  x1      |-->  (q2 + (p3*q2^(1/2))/p4^(1/2))/(p1*p2)
  x2      |-->  p1
  x3      |-->  q2^(1/2)/p4^(1/2)
```

```
  x4      |-->  q2/p5
  x5_hat  |-->  q2/p4
```

and

```
  vbar(1) = q2 + (p3*q2^(1/2))/p4^(1/2)
  vbar(2) = (p3*q2^(1/2))/p4^(1/2)
  vbar(3) = q2
  vbar(4) = q2
  vbar(5) = q2
```

We may now proceed with the inverse substitution.

The mapping function psi_q^-1 is given by

```
  q2  |-->  y5
```

The composite inverse map psi_qp^-1:

```
  p1  |-->  x2
  p2  |-->  k1
  p3  |-->  k2
  p4  |-->  k3
  p5  |-->  k5
  q2  |-->  k4
```

The complete steady state map psi_ss is therefore

```
  k1      |-->  k1
  k2      |-->  k2
  k3      |-->  k3
  k4      |-->  k4
  k5      |-->  k5
  x1      |-->  (k4 + (k2*k4^(1/2))/k3^(1/2))/(k1*x2)
  x2      |-->  x2
  x3      |-->  k4^(1/2)/k3^(1/2)
  x4      |-->  k4/k5
  x5_hat  |-->  k4/k3
```

The unsubstituted steady state reaction velocity vector vbar = psi_ss(v) is given by

```
  vbar(1) = k4 + (k2*k4^(1/2))/k3^(1/2)
  vbar(2) = (k2*k4^(1/2))/k3^(1/2)
  vbar(3) = k4
  vbar(4) = k4
  vbar(5) = k4
```

Verify steady state. xdot = S * vbar, where

```
  xdot(1) = 0
  xdot(2) = 0
  xdot(3) = 0
  xdot(4) = 0
```